# An innovative approach to assessing a first year undergraduate mathematics module

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## Abstract
It is widely acknowledged that many students find the step up from A Level to university mathematics a challenging one. This paper describes a novel type of module that aims to help first year undergraduates bridge the gap by stimulating their interest and enthusiasm, and broadening their knowledge and understanding of mathematics, rather than just teaching them more of it. It has a historical and philosophical flavour that stresses the practical origins and applications of mathematics. It encourages private study and self-sufficiency, areas that new undergraduates often find challenging. A different kind of module demands a different type of assessment; a computer marked test is used, but students are allowed free access to any resources they want, including the internet, course materials, books and notes. It rewards individual research and preparation, and the ability to carry out research under pressure, as well as knowledge and understanding of the module content.

### Keywords
Mathematics education; innovation; assessment; computer marking; historical case studies

## 1. Introduction
There are several reasons why some students find university mathematics difficult compared with A level: it is more difficult; it is taught in a more abstract fashion; and they are expected to be more self-sufficient than they are used to. Another perhaps less recognised problem is that many students do not find it very interesting. They do not see its relevance, they do not see how they might use it and they do not find it very interesting for its own sake. The majority of mathematics undergraduates at the University of Leicester become accountants, managers, teachers and so on; only a small minority become academic mathematicians. For the majority, learning mathematics is a means to an end, not an end in itself. They need to know the mathematics necessary for their future careers but there can be no supposition that they will automatically find all mathematics interesting for its own sake. Yet that is what mathematics professionals usually assume. For example, in explaining the rationale for the choice of topics in *Mathematics: a Very Short Introduction*, Gowers (2002) says he "avoided topics such as chaos theory and Gödel's theorem, which have a hold on the public imagination out of proportion to their impact on current mathematical research" (pp x-xi). He does add that these "are in any case well treated in many other books" (*loc cit*) but the message is: if you want to know what mathematics is about you need to know what current

mathematicians are working on. Is that always the case, though? Might some students respond better to something they regard as more interesting, or useful, even if it is no longer at the cutting edge of research?

Even when students find topics interesting, they can be put off by teaching that concentrates on the mechanics of mathematics rather than its use, which can make it tedious rather than stimulating. Lockhart (2008) puts this very strongly: "… if I had to design a mechanism for the express purpose of destroying a child's natural curiosity and love of pattern-making, I couldn't possibly do as good a job as is currently being done – I simply wouldn't have the imagination to come up with the kind of senseless, soulcrushing ideas that constitute contemporary mathematics education" (p 2). Lockhart's objection is that the teaching of mathematics places all the emphasis on formulae and definitions and none on solving problems. He is writing about American High Schools but similar criticisms can be levelled at the British A level, with the consequence that a perhaps surprising number of students of mathematics arrive at university without any great enthusiasm for the subject.

## 2. The *Mathematics and Society* Module

The University of Leicester has developed a new first year module to help students adjust to university mathematics. It covers topics that students tend to enjoy in order to stimulate their interest and enthusiasm; and the mathematics, while not too demanding, is wide ranging and broadens their knowledge of what it can do. Above all, the module stresses that there is a point to mathematics (*why* we do it, rather than *how*); that it is used for doing useful things in the real world and that is where, ultimately, it all comes from. To stress the point that mathematics is not something divorced from its environment, the module is called *Mathematics and Society*. It is subtitled: *What is the point of simultaneous equations?*[1], which is the sort of question that professional mathematicians often find very hard to answer. For them the point of simultaneous equations is so obvious it does not need an explanation[2] but it is difficult to see why you should expect someone to be interested in mathematics if you cannot explain what it is *for*.

The module consists of nine one-hour lectures, based on historical case studies. The lectures only cover the material briefly; students are encouraged to read more in their own time and need to do so if they are to do well in the assessment. The topics, which will be familiar to those who have taught mathematics through its history, were deliberately chosen to appeal to students. Chaos theory, Gödel's theorems and other popular topics such as the Monty Hall problem and cryptography all appear, along with less obvious areas such as the chemistry of brewing beer. There is an optional field trip to Bletchley Park, to see the work of the code-breakers and hear about the mathematics behind the Enigma coding machine from specialists, which supplements the lecture on cryptography.

To stress the practical origins of mathematics, the breadth of its applications and the evolutionary nature of its development, lectures usually begin with an apparently non-mathematical subject, such as Italian renaissance art, an apple falling from a tree or weather forecasting, before moving on to the mathematics of that subject and how it is used. Of course, while mathematics has its origins in practical applications, once discovered it has a

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1 From a radio item on the failings of mathematics educations at school; it interviewed a bright but disaffected young man who did not enjoy mathematics at school, and asked him why not. He said: "They taught us simultaneous equations; what's the point of simultaneous equations?"

2 This is not a problem unique to mathematicians, of course; most professionals assume that lay people find their subject as important and as interesting as they do.

mind of its own; mathematical structures developed for practical purposes are extended, often in unexpected ways, giving rise to pure mathematics. Hence, some of the lectures explain how aspects of pure mathematics evolve but stress that the purest forms of mathematics can ultimately find practical applications in unforeseen ways. The role of developments in technology in enabling mathematical progress is also included.

The topics are not in any way dumbed down. A wide variety of non-mathematical topics is introduced, and students are expected to refer to the further reading for more information on subjects such as art, science, engineering, medicine, law and internet security. Stress is placed on the underlying mathematics but it is not always possible to go through it in any detail in the lecture, so students are left to ensure in their own time that they understand it. They are then questioned on it in the assessment. The historical and philosophical aspects of the mathematics are incorporated whenever possible, to give further breadth.

## 3. Assessment

Methods of assessment in mathematics tend to be relatively standard. A recent study by Iannone and Simpson (2011) showed that "closed book examination (CBE) constitutes the dominant mode of assessment across almost all of the [sample of] universities" (p 8). The exceptions were in specific areas. For example, "Many departments assessed statistics using open book exams (where the material available to students varies from formula sheets to pre-released set of data) and projects" (*loc cit*); while in history of mathematics and mathematics education modules "assessment tended to be through essays, projects and presentations, with far less reliance on closed book examinations" (*loc cit*). None of these methods of assessment seemed appropriate for *Mathematics and Society*, where the requirements are that it encourages individual research, preparation and ability to work under pressure, in addition to knowledge and understanding of the module content. Instead, a computer-marked test is used but students are allowed free access to any resources they want, including the internet, course materials, books and notes. The only restriction is that they cannot communicate with each other. The main constraint on students' performance is then time, so it is vital that the number and difficulty of the questions corresponds to the time available.

The advantage of this type of assessment is that the questions are no longer restricted to the material in the lectures and so can range more widely over related material than would usually be the case. Students are aware of this and must mitigate it by extensive preparation before the event. The questions concentrate much more on the mathematics (or its history or philosophy) than the lectures and where possible require a significant understanding of the mathematical principles involved.

The assessment uses the test facility in *Blackboard*, the University's virtual learning environment. Blackboard provides for many different types of question and several are utilised[3] in order to provide variety and avoid some of the drawbacks of straight multiple choice tests[4]. There are 36 questions (144 marks) in total and the time allowed is three hours[5]. Students are given their marks at the end of the test. The difficulty of the questions

[3] Multiple choice; multiple answer; fill in the blank(s); numerical response (where the answer is a number, within a specified margin of error); matching and hot spot (where the student is asked to identify a point on a picture or diagram).

[4] For an account of some of these, see, for example, Kantrov (2000).

[5] This was in itself an innovation for students, most of whom had not previously sat examinations lasting more than two hours. Announcement of the duration was generally met be an incredulous "three *hours*" but there were few complaints after the test and those were mainly that there had not been *enough* time.

varies greatly: some are quite easy (though there is no question that all students get right); others rather hard (though there is no question that everyone gets wrong). If necessary, the overall difficulty of the assessment could be changed, for example by replacing multiple choice with numerical response questions, which tend to be harder, or vice versa.

Creating meaningful questions that can be computer-marked when students have access to the internet is quite challenging. If all the answers can be found in this way then there is not much point to the test, but so much material is available on the internet that it is hard to avoid. However, it is not necessary to exclude Google completely because it encourages students to prepare by researching useful web sites beforehand. That they do so could be seen by watching their use of the internet during the test.

To give students an idea of what to expect, sample questions are made available during the module. Copies of the presentations accompanied by extensive notes are handed out at the end of each lecture and made available on Blackboard, as an aid to preparation. Students are also given additional papers on specific topics they might not be familiar with but the only other materials provided are reading lists and links to useful web sites, with the indication that the student must explore these if they are to do well in the assessment.

To date, the outcomes have demonstrated that the test can provide a valid means of assessment. The 2011/2 results (65 students) gave a good spread of marks:

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| 1st | 26.2% |
| 2.1 | 40.0% |
| 2.2 | 20.0% |
| 3rd | 12.3% |
| Fail | 1.5% |

Individual scores ranged from 36.5% to 84%, with an average of 62.6%, and showed reasonable correlation with marks in other, more traditional modules. The results for individual questions also showed a good spread; the range was 12.3% to 98.5%.

## 4. The Views of Students

As this was a new module, and of a rather unusual type, the 2011/12 cohort was asked to complete a detailed questionnaire, asking students how much they enjoyed the module, how worthwhile they thought it was, what they had learned and what they thought could be improved. Completing it was not compulsory and responses were submitted anonymously but students were encouraged to respond, be honest and be constructive. 25 students (38.5%) responded. Whether the rest did not have strong feelings or did but could not be bothered is not clear but the results should clearly be treated with some caution.

Those students who did respond liked the module, in terms of both style and content; 88% said they liked the module enormously or quite a lot. They found it interesting and different, with clear evidence reinforcing the view that mathematics students do not necessarily find the contents of a typical university mathematics curriculum very stimulating. They liked the range of subjects and the fact it did not concentrate on formulae or calculations, and they enjoyed the historical aspects of the module and seeing how mathematics relates to the society in which it is found.

Most students found the module worthwhile – 80% said it was very worthwhile or quite useful – and no one thought it a waste of time. They said it introduced them to aspects of mathematics they might not otherwise have come across and most mentioned specific topics. More generally, they said it gave them a wider understanding of the subject, illustrated the range of applications of mathematics, made the mathematics they studied elsewhere more relevant, and showed how it relates to the society in which it is found. Some also mentioned the field trip. There were few negative comments but some students were not sure how *useful* it was or whether they would need any of it in the future[6].

Most students did not think the module needed improving. Of those that offered suggestions the most common issue was the examination; it was well liked and thought fair but some wanted more time (which would rather defeat the object) or the test split in two. Several students expressed a desire for practical exercises on the topics. The only other topic to attract more than one comment was for more areas of mathematics to be covered.

## 5. Concluding Remarks

The experience of this module supports the contention that a module like *Mathematics and Society* can play a valuable role in the first year undergraduate mathematics curriculum. It appears to succeed in its primary objectives of increasing students' interest, stimulating their enthusiasm and broadening their knowledge (both mathematical and general). They become aware of aspects of mathematics they might not otherwise have known about, including historical and (to a lesser extent) philosophical aspects of the subject, and do seem to pursue them further. Students like the module, some of them comparing it favourably to straight mathematics, and generally see it as worthwhile. The case study style goes down well and most of the topics seem to engage their interest.

The experience also shows that a computer-marked examination in which students have access to external resources can be a useful addition to the lecturer's armoury. It encourages students to work independently; it rewards preparation and the ability to research under time pressure. Students like this form of examination, which is considered challenging but fair and hard work but fun, and they like the fact that they are given their marks immediately. Whether the module can contribute to an overall improvement in students' results remains to be seen.

## 6. References


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[6] It would be interesting to know whether they thought the same about their other modules.