Lecture 5 - Feedforward

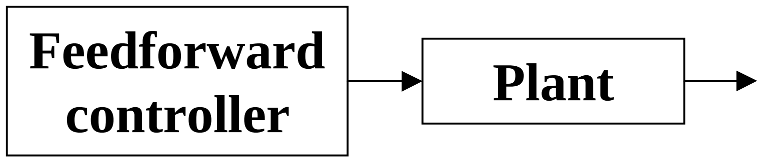
- Programmed control
- Path planning and nominal trajectory feedforward
- Feedforward of the disturbance
- Reference feedforward, 2-DOF architecture
- Non-causal inversion
- Input shaping, flexible system control
- Iterative update of feedforward

Why Feedforward?

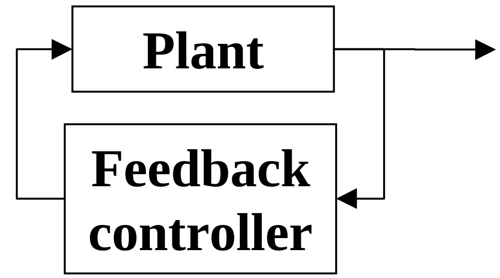
- Feedback works even if we know little about the plant dynamics and disturbances
- Was the case in many of the first control systems
- Much attention to feedback - for historical reasons

- Open-loop control/feedforward is increasingly used
- Model-based design means we know something
- The performance can be greatly improved by adding open-loop control based on our system knowledge (models)

Feedforward

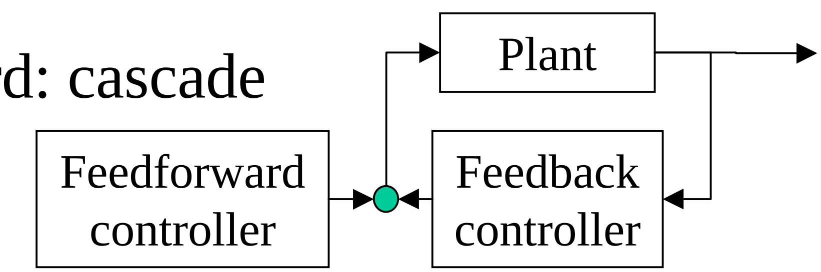


– this Lecture 5



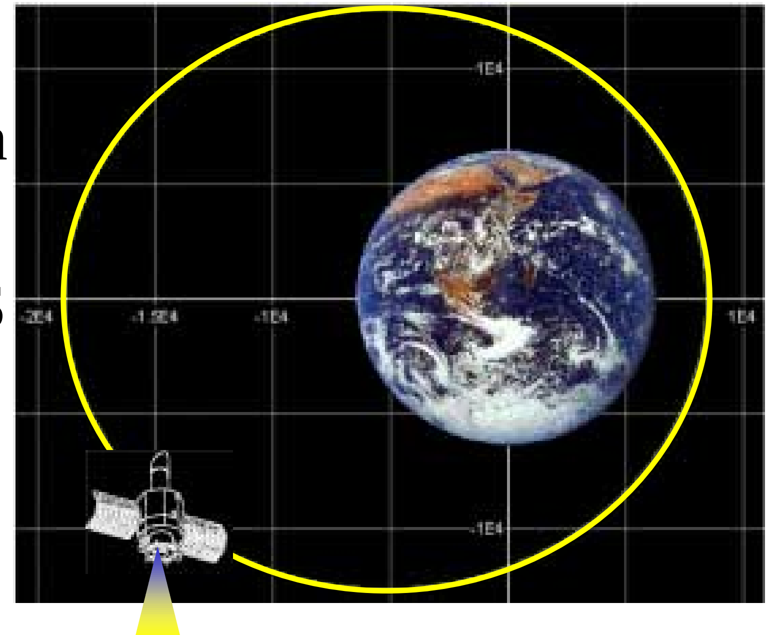
– Lecture 4 PID
– Lecture 6 Analysis
– Lecture 7 Design

- Main premise of the feedforward control: a model of the plant is known
- Model-based design of feedback control - the same premise
- The difference: feedback control is less sensitive to modeling error
- Common use of the feedforward: cascade with feedback



Open-loop (programmed) control

- Control $u(t)$ found by solving an optimization problem. Constraints on control and state variables.
- Used in space, missiles, aircraft FMS
  - Mission planning
  - Complemented by feedback corrections
- Sophisticated mathematical methods were developed in the 60s to overcome computing limitations.
- Lecture 12 will get into more detail of control program optimization.



$$\dot{x} = f(x,u,t)$$

$$J(x,u,t) \rightarrow \min$$

$$x \in \mathbf{X}, u \in \mathbf{U}$$

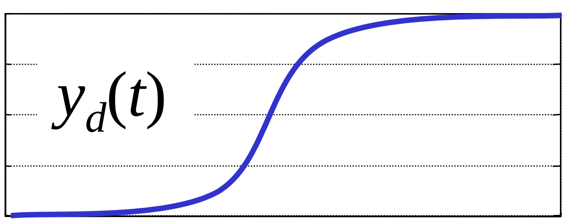
Optimal control: $u = u_*(t)$

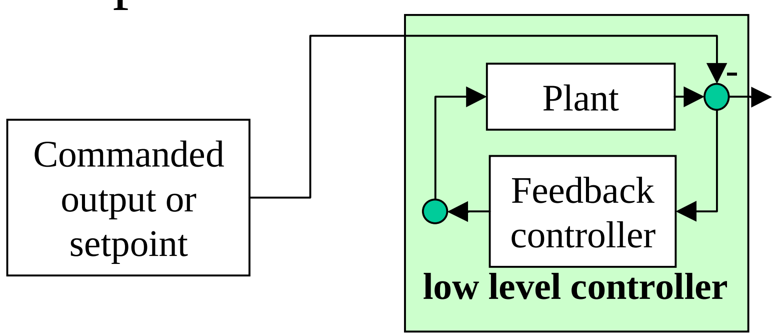
Optimal control

- Performance index and constraints
- Programmed control
  - compute optimal control as a time function for particular initial (and final) conditions
- Optimal control synthesis
  - find optimal control for *any* initial conditions
  - at any point in time apply control that is optimal now, based on the current state. This is *feedback* control!
  - example: LQG for linear systems, gaussian noise, quadratic performance index. Analytically solvable problem.
  - simplified model, toy problems, conceptual building block
- MPC - will discuss in Lecture 12

Path/trajectory planning

- The disturbance caused by the change of the command *r* influences the feedback loop.
- The error sensitivity to the reference $R(s)$ is bandpass: $|R(i\omega)| << 1$ for ω small
- A practical approach: choose the setpoint command (path) as a smooth function that has no/little high-frequency components. No feedforward is used.
- The smooth function can be a spline function etc

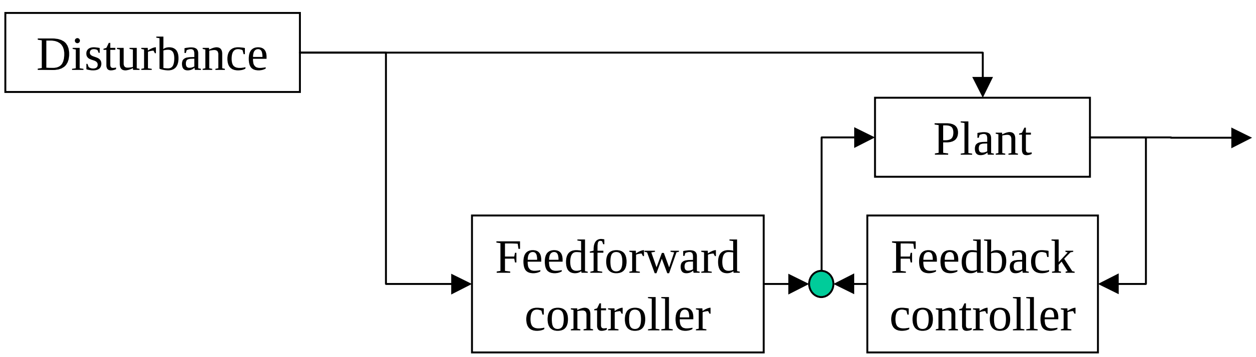




Disturbance feedforward

- Disturbance acting on the plant is measured
- Feedforward controller can react *before* the effect of the disturbance shows up in the plant output

Example:
Temperature control. Measure ambient temperature and adjust heating/cooling
• homes and buildings
• district heating
• industrial processes - crystallization
• electronic or optical components

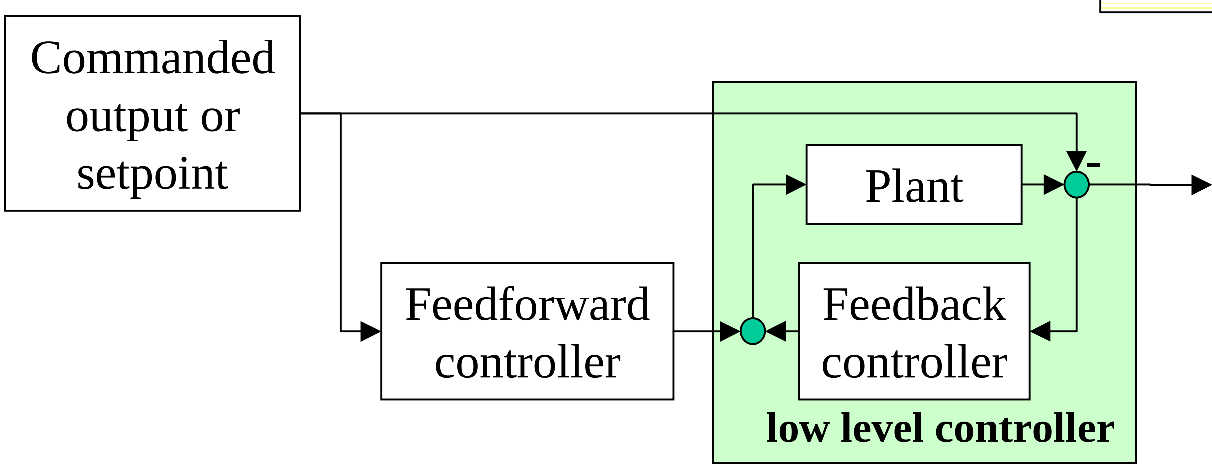


Command/setpoint feedforward

- The setpoint change acts as disturbance on the feedback loop.
- This disturbance can be measured
- 2-DOF controller

Examples:

•Servosystems
– robotics
•Process control
– RTP
•Automotive
– engine torque demand



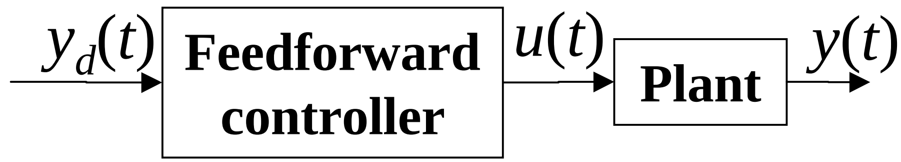
Feedforward as system inversion

$$y = P(s)u$$

$$y = y_d \Rightarrow u = [P(s)]^{-1} y_d$$

$$e = P(s)u + D(s)d$$

$$y_d \equiv -D(s)d$$



- Simple example:

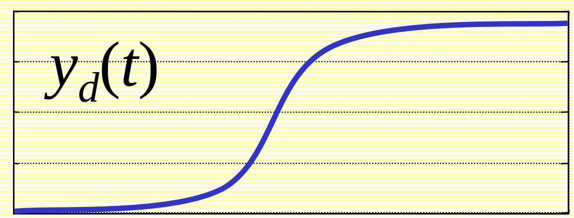
$$P(s) = \frac{1+2s}{1+s}$$

$$[P(s)]^{-1} = \frac{1+s}{1+2s}$$

More examples:

•Disk drive long seek

•Robotics: tracking a trajectory



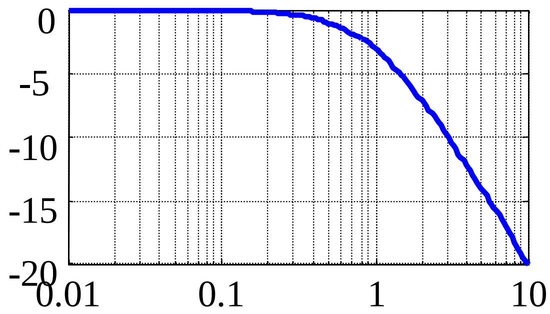
Feedforward as system inversion

$$y = P(s)u$$

$$y = y_d \Rightarrow u = [P(s)]^{-1} y_d$$

$$\tilde{u}(i\omega) = \frac{\tilde{y}_d(i\omega)}{P(i\omega)}$$

- Issue
  - high-frequency roll-off



$$P(s) = \frac{1}{1+s}$$ proper

$$[P(s)]^{-1} = 1 + s$$ non-proper

- Approximate inverse solution:
  - ignore high frequency in some way

Proper transfer functions

- Proper means deg(Denominator) ≥ deg(Numerator)
- Strictly proper <=> high-frequency roll-off, all physical dynamical systems are like that
- Proper = strictly proper + feedthrough
- State space models are always proper
- Exact differentiation is noncausal, non-proper
- Acceleration measurement example

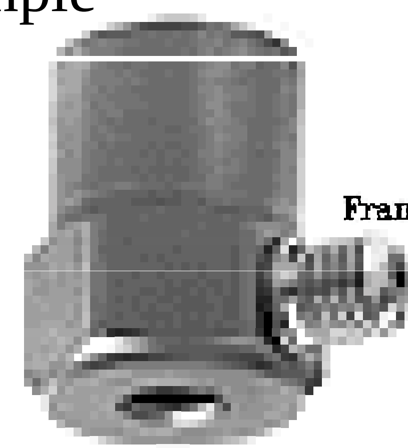
$$m\ddot{x} = u \qquad a = \ddot{x}$$

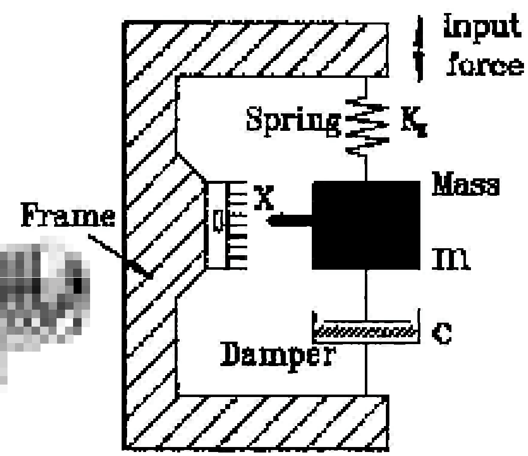
$$u = ma - k(x - x_d)$$

$$\Rightarrow x = x_d$$

this is wrong!

accelerometer





Differentiation

- Path/trajectory planning - mechanical servosystems
- The derivative can be computed if $y_d(t)$ is known ahead of time (no need to be causal then).

$$P^{-1}(s)y_d = \frac{1}{P(s)} \cdot \frac{1}{s^n} y_d^{[n]}, \qquad y_d^{[n]}(t) = \frac{d^n y}{dt^n}(t)$$

$$P(s) = \frac{1}{1+s}$$

$$P^{-1}(s)y_d = \frac{1+s}{s}\dot{y}_d = \left(1 + \frac{1}{s}\right)\dot{y}_d = \dot{y}_d + y_d$$

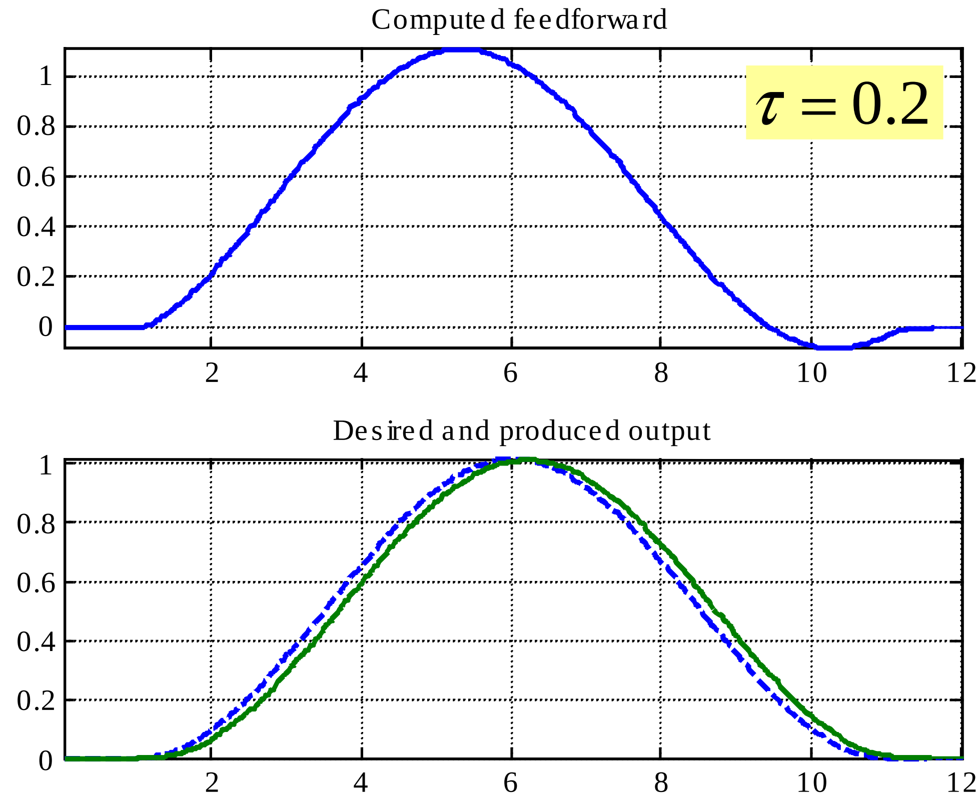
Approximate Differentiation

- Add low pass filtering:

$$P^{\dagger}(s) = \frac{1}{(1+\tau s)^n} \cdot \frac{1}{P(s)}$$

$$P(s) = \frac{1}{1+s}$$

$$P^{\dagger}(s) = \frac{1}{1+\tau s} \cdot (1+s)$$



‘Unstable’ zeros

- Nonminimum phase system
  - r.h.p. zeros $\rightarrow$ r.h.p. poles
  - approximate solution: replace r.h.p. zeros by l.h.p. zeros

$$P(s) = \frac{1-s}{1+0.25s}, \qquad P^{\dagger}(s) = \frac{1+0.25s}{1+s}$$

- RHP zeros might be used to approximate dead time
  - exact causal inversion impossible

$$P(s) = e^{-2Ts} \approx \frac{1-sT}{1+sT}$$

- If preview is available, use a lead to compensate for the deadtime

Two sided *z*-transform, non-causal system

- Linear system is defined by a pulse response. Do not constrain ourselves with a causal pulse response anymore

$$y(x) = \sum_{k=-\infty}^{\infty} h(x-k)u(k)$$

- 2-sided z-transform gives a “transfer function”

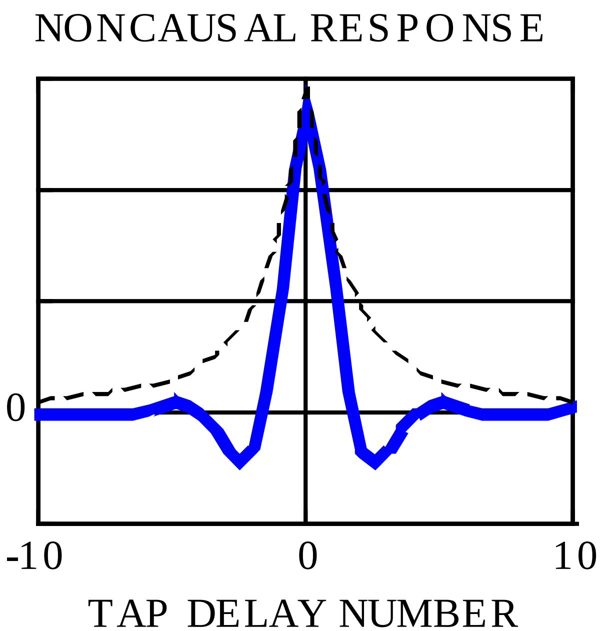
$$P(z) = \sum_{k=-\infty}^{\infty} h(k)z^{-k}$$

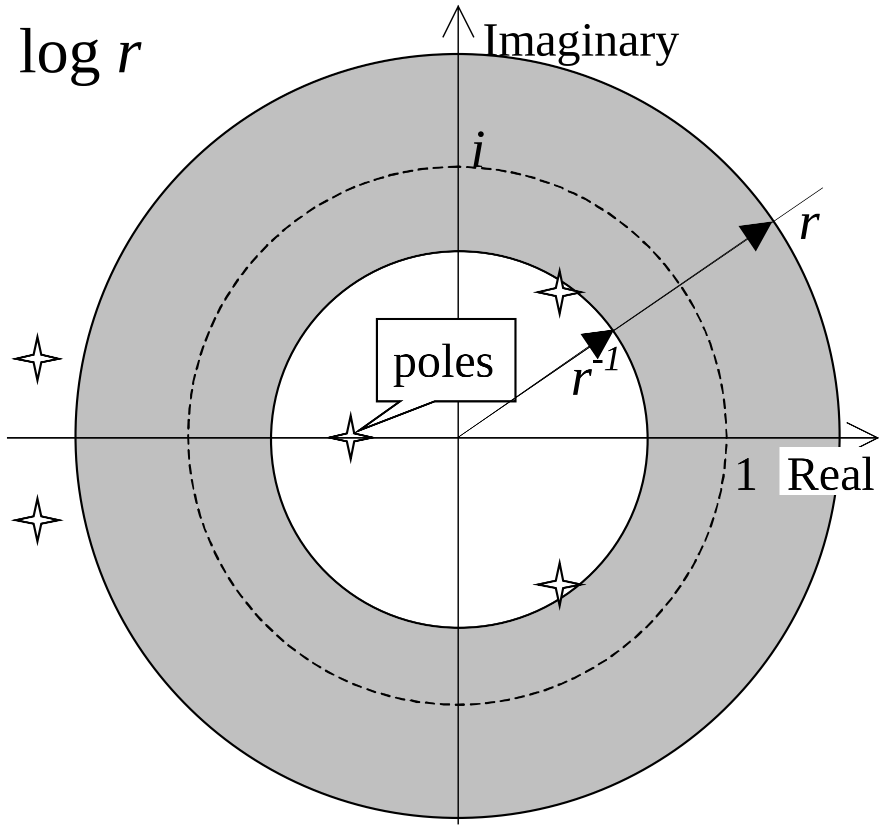
- Fourier transform/Inverse Fourier transform are two-sided

• Oppenheim, Schafer, and Buck, *Discrete-Time Signal Processing*, 2nd Edition, Prentice Hall, 1999.

Impulse response decay

- Decay rate from the center = $\log r$



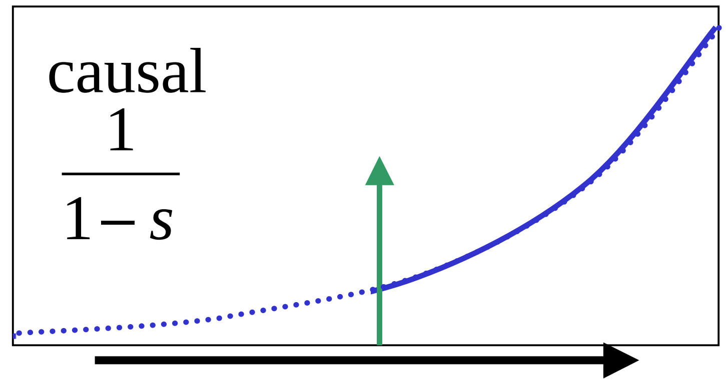


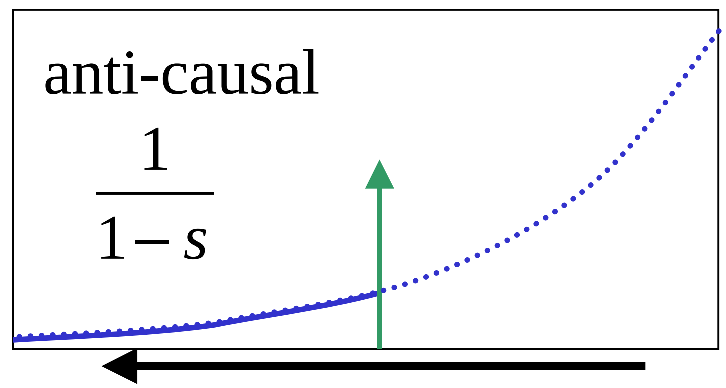
Non-causal inversion

- Causal/anti-causal decomposition
  - 2-sided Laplace-transform

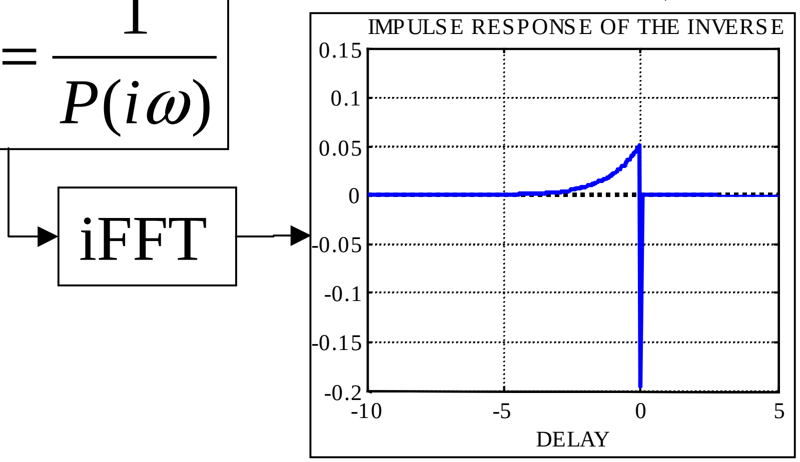
$$P(s) = \frac{1-s}{1+0.25s}$$

$$P^{-1}(s) = \frac{1+0.25s}{1-s} = -0.25 + \underbrace{\frac{1.25}{1-s}}_{\leftarrow}$$





$$P^{-1}(i\omega) = \frac{1}{P(i\omega)}$$



Frequency domain inversion

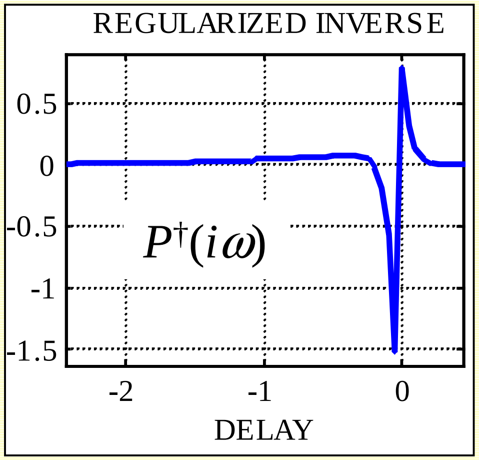
- Regularized inversion: $\|y_d - Pu\|_2^2 + \rho\|u\|_2^2 \to \min$

$$\int\left(\left|y_d(i\omega) - P(i\omega)u(i\omega)\right|^2 + \rho\left|u(i\omega)\right|^2\right)d\omega \to \min$$

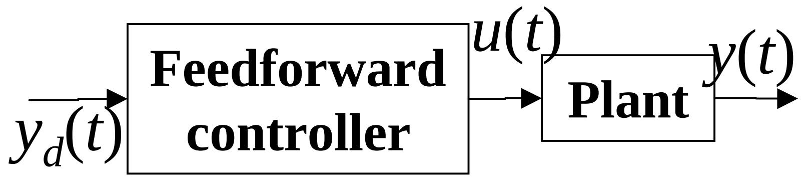
$$u(i\omega) = \frac{P^*(i\omega)}{P^*(i\omega)P(i\omega) + \rho}\, y_d(i\omega) = P^\dagger(i\omega)\, y_d(i\omega)$$

- Systematic solution
  - simple, use FFT
  - takes care of everything
  - noncausal inverse
  - high-frequency roll-off
  - Paden & Bayo, 1985(?)

$$P(s) = \frac{1-s}{(1+0.25s)(1+s)}$$

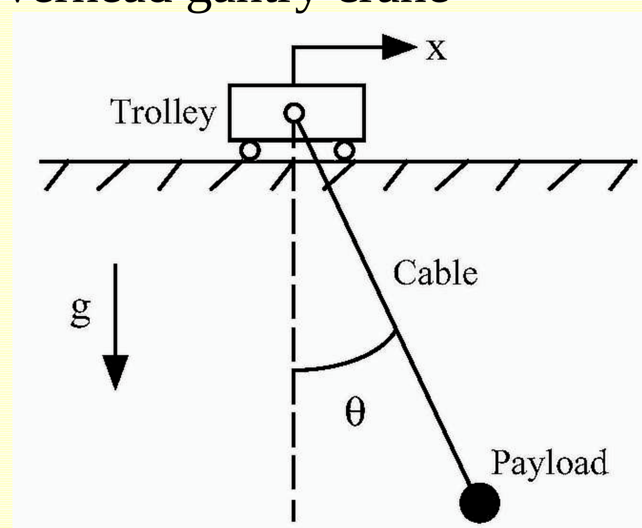


Input Shaping: point-to-point control



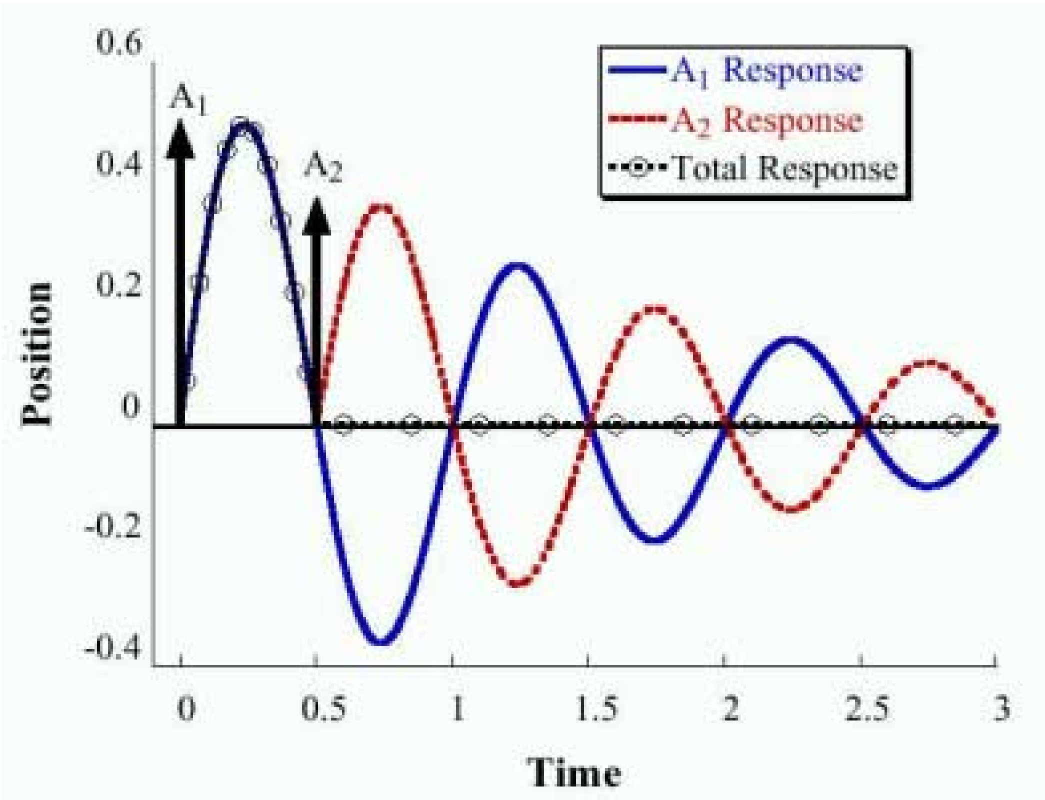
- Given initial and final conditions find control input
- No intermediate trajectory constraints
- Lightly damped, imaginary axis poles
  - preview control does not work
  - other inversion methods do not work well
- FIR notch fliter
  - Seering and Singer, MIT
  - Convolve Inc.

Examples:
- Disk drive long seek
- Flexible space structures
- Overhead gantry crane



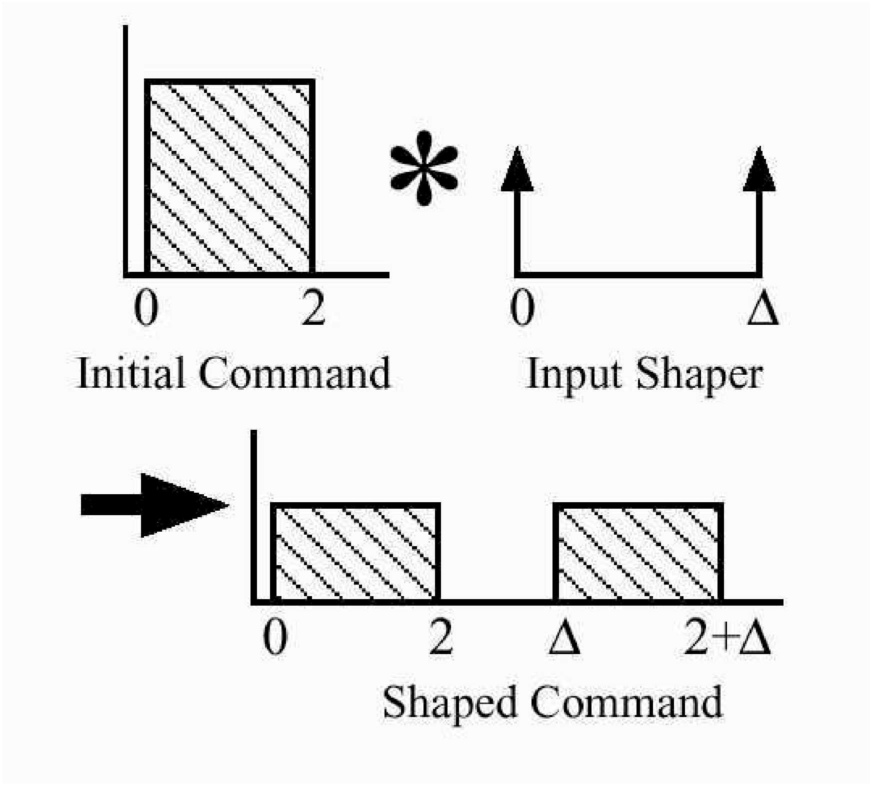
Pulse Inputs

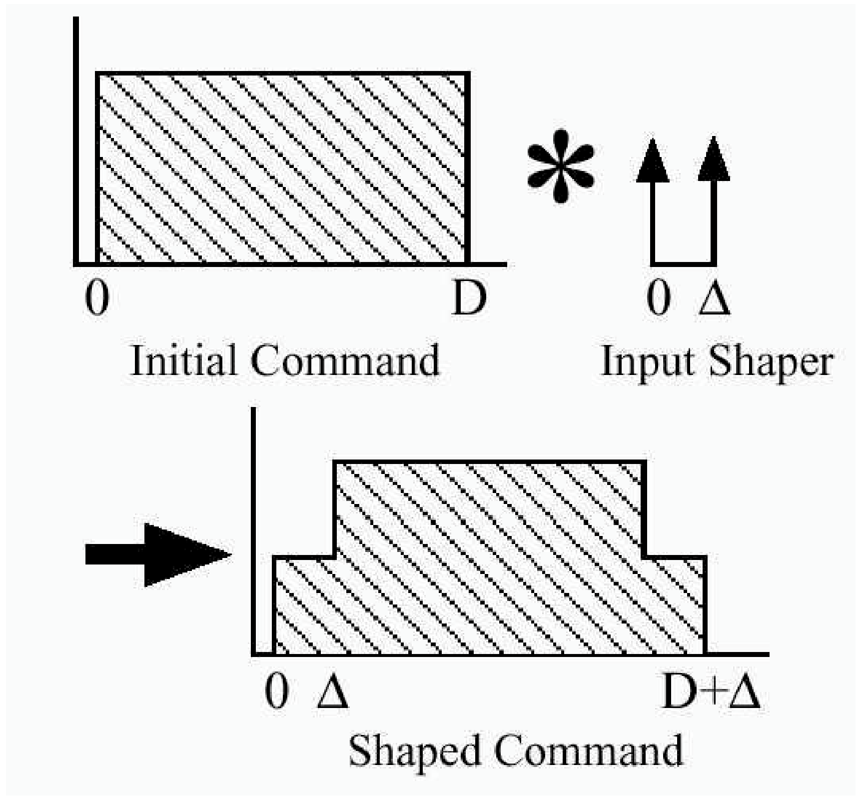
- Compute pulse inputs such that there is no vibration.
- Works for a pulse sequence input
- Can be generalized to *any* input



Input Shaping as signal convolution

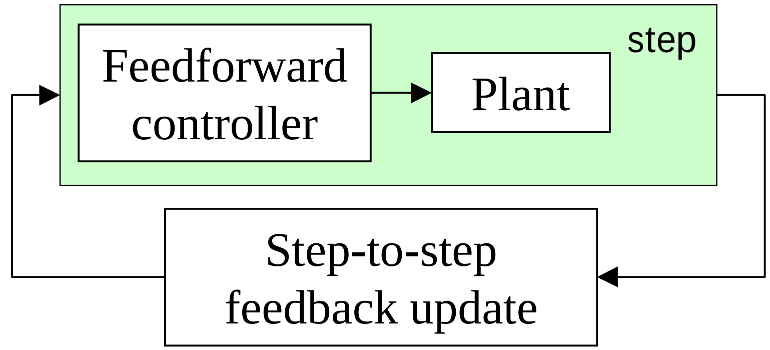
- Convolution: $f(t) * \left(\sum A_i \delta(t - t_i)\right) = \sum A_i f(t - t_i)$



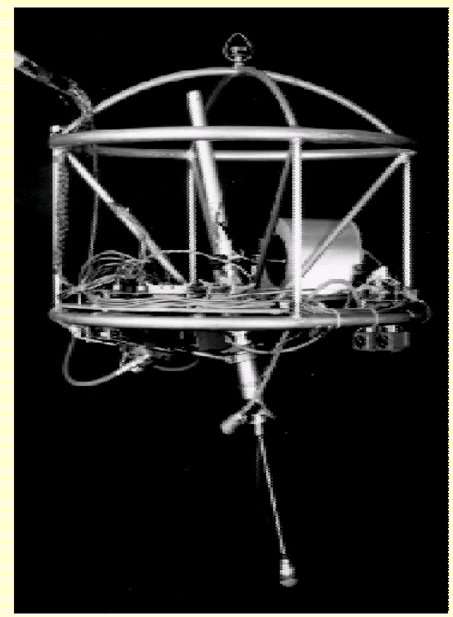


Iterative update of feedforward

- Repetition of control tasks
- Robotics
  – Trajectory control tasks: Iterative Learning Control
  – Locomotion: steps
- Batch process control
  – Run-to-run control in semiconductor manufacturing
  – Iterative Learning Control (*IEEE Control System Magazine*, Dec. 2002)



Example:
One-legged hopping machine (M.Raibert)



Height control:

$y_d = y_d(t-T_n;a)$

$h(n+1)=h(n)+Ga$

Feedforward Implementation

- Constraints and optimality conditions known ahead of time
  - programmed control
- Disturbance feedforward in process control
  - has to be causal, system inversion
- Setpoint change, trajectory tracking
  - smooth trajectory, do not excite the output error
  - in some cases have to use causal ‘system inversion’
  - preview might be available from higher layers of control system, noncausal inverse
- Only final state is important, special case of inputs
  - input shaping - notch filter
  - noncausal parameter optimization

Feedforward Implementation

- Iterative update
  - ILC
  - run-to-run
  - repetitive dynamics

- Replay pre-computed sequences
  - look-up tables, maps
- Not discussed, but used in practice
  - Servomechanism, disturbance model
  - Sinusoidal disturbance tracking - PLL
  - Adaptive feedforward, LMS update